Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

FURTHER MATHEMATICS **9231/21**

Paper 2 **May/June 2017**

MARK SCHEME

Maximum Mark: 100

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
SOI	Seen or implied
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1	$0{\cdot}08 \times (300 - v) = 1000 \times 0{\cdot}02$ (AEF)	**M1 A1**	Find eqn for exit speed v from e.g. change in momentum = Ft (if $300 + v$ or equivalent, can allow **M1** only)
	$v = 300 - 250 = 50$ [m s^{-1}]	**A1**	
	Total:	**3**	
2	Take moments for rod about some point such as: A: $R_P \times AP - kW \times 3a \cos\theta = W \times (3a/2) \cos\theta$ [$R_P \times 3a/4 - kW \times 9a/5 = W \times 9a/10$ so $15\, R_P - 36\, kW = 18\, W$] O: $F_A \times (5a/4) - kW \times (3a \cos\theta - 5a/4)$ $= - W \times (5a/4 - (3a/2) \cos\theta)$ [$F_A \times 5a/4 - kW \times 11a/20 = - W \times 7a/20$ so $25\, F_A - 11\, kW = -7\, W$] P: $R_A \times AP \sin\theta + F_A \times AP \cos\theta - kW \times (3a - AP) \cos\theta$ $= W \times (3a/2 - AP) \cos\theta$) [$R_A \times 3a/5 + F_A \times 9a/20 - kW \times 27a/20 = W \times 9a/20$ so $12\, R_A + 9\, F_A - 27\, kW = 9\, W$] B: $R_P \times (3a - AP) - R_A \times 3a \sin\theta - F_A \times 3a \cos\theta$ $= W \times (3a/2) \cos\theta$ [$R_P \times 9a/4 - R_A \times 12a/5 - F_A \times 9a/5 = W \times 9a/10$ so $45\, R_P - 48\, R_A - 36\, F_A = 18\, W$] C: $R_P \times (3a/2 - AP) - R_A \times (3a/2) \sin\theta - F_A \times (3a/2) \cos\theta + kW \times (3a/2) \cos\theta = 0$ [$R_P \times 3a/4 - R_A \times 6a/5 - F_A \times 9a/10 + kW \times 9a/10 = 0$ so $15\, R_P - 24\, R_A - 18\, F_A + 18\, kW = 0$] F: $R_P \cos\theta \times (3a - AP) \cos\theta - R_P \sin\theta \times AP \sin\theta$ $- F_A \times 3a \cos\theta = W \times (3a/2) \cos\theta$ [$(3/5)R_P \times 27a/20 - (4/5)R_P \times 3a/5$ $- F_A \times 9a/5 = W \times 9a/10$ so $81\, R_P - 48\, R_P - 180\, F_A = 90\, W$]	**M1 A1**	F_A here denotes friction on rod measured in downward dirn; P denotes point of contact of rod and disc; θ denotes angle between rod and horizontal. [$AP = 3a/4$, $\sin\theta = 4/5$, $\cos\theta = 3/5$, $\tan\theta = 4/3$, $3a - AP = 9a/4$, $3a/2 - AP = 3a/4$] See note below on solving question without introducing R_P (C denotes mid-point of AB) (F is vertically below B, on AO extended)

Question	Answer	Marks	Guidance
	Find two more indep. eqns, e.g. resolution of forces on rod: Horizontally: $R_A = R_P \sin\theta\,[= 4R_P/5]$ Vertically: $F_A + (k+1)W = R_P \cos\theta\,[= 3R_P/5]$ Along AB: $R_A \cos\theta = F_A \sin\theta + (k+1)W \sin\theta$ Normal to AB: $R_P = R_A \sin\theta + F_A \cos\theta + (k+1)W \cos\theta$	**B1 B1**	A second moment eqn. may be used instead of a resolution Count as 2 eqns if used with moments about P (so R_P absent)
	$F_A = +R_A / 8$ *or* $-R_A / 8$ as appropriate	**B1**	Relate F_A and R_A (may be implied; and must be consistent with friction taken down or up in above eqns)
	[$\sin\theta = 4/5$, $\cos\theta = 3/5$, $\tan\theta = 4/3$]	**M1**	Eliminate θ from all reqd. independent eqns. for forces Find either value of k from reqd. independent eqns. for forces
	$F_A \downarrow$: [$R_P = 6W$, $F_A = 3W/5$, $R_A = 24W/5$], $k = 2$ $F_A \uparrow$: [$R_P = 30W/17$, $F_A = 3W/17$, $R_A = 24W/17$], $k = 4/17$	**M1 A1**	(or 0·235)
	Total:	**8**	

Question	Answer	Marks	Guidance
3(i)	$3mv_A + mv_B = 3mu$, $v_B - v_A = eu$ (AEF)	**M1**	Use momentum <u>and</u> Newton's law (**M0** if inconsistent LHS signs; allow $3v_A + v_B = 3u$)
	$v_A = \frac{1}{4}(3 - e)u$, $v_B = \frac{3}{4}(1 + e)u$	**A1, A1**	Combine to find velocities of A and B after colln. (signs must be consistent with chosen direction)
	Total:	**3**	
3(ii)	$v_B' = -\frac{3}{4} v_B$ $[= -(9/16)(1 + e)u]$ (AEF)	**B1**	Relate velocity v_B' of B after colln. with wall to v_B
	$[3mV_A +]\ mV_B = 3mv_A + mv_B'$ $[V_B = 3(9 - 7e)u/16]$	**M1**	Use momentum (allow m omitted and $V_A = 0$)
	$V_B\ [- V_A] = -e(v_B' - v_A)$ $[V_B = e(21 + 5e)u/16]$	**M1**	Use Newton's law
	EITHER: $[4V_A =]\ (3 - e)v_A + (1 + e)v_B' = 0$ $\frac{1}{4}(3 - e)^2 - (9/16)(1 + e)^2 = 0$ (AEF)	**(M1 A1)**	Eliminate V_B with $V_A = 0$ and substitute for v_A and v_B'
	OR: $3(9 - 7e) = e(21 + 5e)$	**(M1 A1)**	
	$5e^2 + 42e - 27 = 0$, $e = 3/5$ *or* $0{\cdot}6$	**M1 A1**	Form and solve quadratic for e, rejecting root –9
	Total:	**7**	

Question	Answer	Marks	Guidance
4(i)	$I_{discs} = ½\, ma^2 + 2 \times \{½\, ma^2 + m\,(4a)^2\}$ $[= (½ + 2 \times \{33/2)\}\, ma^2 = 67\, ma^2/2]$	**M1 A1**	Find MI of discs about axis l
	I_{AB} *or* $I_{AC} = ⅓(⅓m)\, a^2 + (⅓m)\,(2a)^2$ (AEF) $[= 13\, ma^2/9]$	**M1 A1**	Find MI of e.g. rod joining one of A,B or A,C about axis l (**M1** for finding MI of any of the 3 rods)
	$I_{BC} = ⅓(⅓m)\, a^2 + (⅓m)\,(2a\sqrt{3})^2$ (AEF) $[= 37\, ma^2/9]$	**A1**	Find MI of rod joining B,C about axis l
	$I = (67/2 + 37/9 + 2 \times 13/9)\, ma^2 = 81\, ma^2/2$	**A1**	Combine to find MI of object about axis l
	Total:	**6**	
4(ii)	$h = 4a$	**B1**	Find or state vertical change h of centre of mass
	$½\, I\, \omega^2 = 4\, mgh,\ \omega^2 = 64g\,/81a$	**M1 A1 FT**	Find angular velocity ω when B below A by energy (FT on I)
	$\omega = (8/9)\,\sqrt{(g/a)}$ *or* $0{\cdot}889\,\sqrt{(g/a)}$ *or* $2{\cdot}81/\sqrt{a}$	**A1**	(requires some simplification for this **A1**)
	Total:	**4**	
5(i)	$½mv_1^2 = ½mu^2 + mga \cos \alpha$ $v_1^2 = ag + 2\, ag \cos \alpha,\ v_1 = \sqrt{(ag\,(1 + 2 \cos \alpha))}$ AG	**M1 A1**	Verify v_1 for string horizontal by consvn of energy (**A0** if no m)
	Total:	**2**	

Question	Answer	Marks	Guidance
5(ii)	$T_A + mg\cos\alpha = m(\sqrt{ag})^2/a$, $T_A = mg(1 - \cos\alpha)$ $\frac{1}{2}mv_2^2 = \frac{1}{2}mu^2 + mga\cos\alpha - mg\,\frac{2}{3}\,a\cos 60°$	M1 A1	Find tension T_A at A from $F = ma$ radially Find v_2^2 at C by consvn. of energy (**A0** if no m)
	or $\frac{1}{2}mv_1^2 - mg\,\frac{2}{3}\,a\cos 60°$	M1 A1	
	$v_2^2 = ag + 2ag\cos\alpha - \frac{2}{3}ag = ag(\frac{1}{3} + 2\cos\alpha)$	A1	
	$T_C + mg\cos 60° = m\,v_2^2/\frac{2}{3}a$ $[= 3m\,v_2^2/2a]$ $[T_C = 3mg\cos\alpha]$	M1 A1	Find tension T_C at C from $F = ma$ radially
	$mg(1 - \cos\alpha) = 3mg(\frac{1}{3} + 2\cos\alpha)/2 - \frac{1}{2}mg$	M1 A1	Find $\cos\alpha$ from $T_A = T_C$ and substituting for v_2^2
	$1 - \cos\alpha = 3\cos\alpha$, $\cos\alpha = \frac{1}{4}$	A1	
	Total:	**10**	

Question	Answer	Marks	Guidance
6(i)	$P(X \le 4) = 1 - q^4$	**M1**	Find prob. of score of 6 on no more than 4 throws
	$= 671/1296$ *or* $0{\cdot}518$	**A1**	Set $q = 5/6$ and evaluate
	Total:	**2**	
6(ii)	$1 - q^{N-1} > 0{\cdot}95$	**M1**	Formulate condition for N ($1 - q^N$ is **M0**)
	$(5/6)^{N-1} < 0{\cdot}05$, $N - 1 > \log 0{\cdot}05 / \log 5/6$	**M1**	Set $q = 5/6$, rearrange and take logs (any base) to give bound
	$N - 1 > 16{\cdot}4[3]$, $N_{\min} = 18$	**A1**	Find $N_{\min}$ ($N - 1 < 16{\cdot}4$ or $N - 1 = 16{\cdot}4$ earns **M1 M1 A0**)
	Total:	**3**	
7	$\bar{x} = 7{\cdot}2$	**B1**	Find sample mean
	$s^2 = (542 - 72^2/10) / 9$ [$= 118/45$ *or* $2{\cdot}622$ *or* $1{\cdot}619^2$]	**M1**	Estimate population variance (allow biased here: $2{\cdot}36$ *or* $1{\cdot}536^2$)
	H_0: $\mu = 6{\cdot}2$, H_1: $\mu > 6{\cdot}2$ (AEF)	**B1**	State hypotheses (**B0** for $\bar{x}$ ...)
	$t_{9,\,0.95} = 1{\cdot}83[3]$	**B1**	State or use correct tabular t-value
	$t = (\bar{x} - 6{\cdot}2) / (s/\sqrt{10}) = 1{\cdot}95$ [Accept H_1:]	**M1 A1**	Find value of t (or can compare $\bar{x}$ with $6{\cdot}2 + 0{\cdot}939 = 7{\cdot}14$) Consistent conclusion
	Claim (of mean mass increased) is justified (AEF)	**B1 FT**	(FT on both t-values)
	Total:	**7**	

Question	Answer	Marks	Guidance
8(i)	$F(x) = \int f(x)\,dx = x^2/8 - x/4$ [$+ c$]	M1	Find or state distribution function $F(x)$ for $2 \leqslant x \leqslant 4$ using $F(2) = 0$ or $F(4) = 1$ to find c if necessary
	$= x^2/8 - x/4$ *or* $\{(x-1)^2 - 1\}/8$ (AEF)	A1	State $F(x)$ for other values of x
	$F(x) = 0$ $(x < 2)$, $F(x) = 1$ $(x > 4)$	A1	
	Total:	3	
8(ii)	*EITHER:* $G(y) = P(Y < y) = P((X-1)^3 < y)$ $= P(X < 1 + y^{1/3}) = F(1 + y^{1/3})$ $= (1 + y^{1/3})^2/8 - (1 + y^{1/3})/4$ *or* $(y^{2/3} - 1)/8$	(M1 A1)	Find or state $G(y)$ for $2 \leqslant x \leqslant 4$ from $Y = (X-1)^3$ (allow $<$ or $\leqslant$ throughout)
	OR: Use $x = 1 + y^{1/3}$ to find $f(x) = \frac{1}{4} y^{1/3}$ *and* $dx/dy = \frac{1}{3} y^{-2/3}$	(M1 A1)	Find $f(x)$ *and* dx/dy for use in $g(y) = f(x) \times \lvert dx/dy \rvert$
	$g(y)$ [$= G'(y)$] $= (1/12)\, y^{-1/3}$ *or* $1 / (12\, y^{1/3})$	A1	Find $g(y)$ in simplified form
	for $1 \leqslant y \leqslant 27$ [$g(y) = 0$ otherwise]	A1	State corresponding range of y for $G(y)$ *or* $g(y)$
	Total:	4	
8(iii)	$(m^{2/3} - 1)/8 = \frac{1}{2}$	M1	Find median value m of Y from $G(m) = \frac{1}{2}$
	$m^{2/3} = 5$, $m = \sqrt{125}$ *or* $5\sqrt{5}$ *or* $11{\cdot}2$	M1 A1	
	Total:	3	

Question	Answer	Marks	Guidance
9	H_0: $\mu_X = \mu_Y$, H_1: $\mu_X \neq \mu_Y$ (AEF)	**B1**	State hypotheses (**B0** for $\bar{x}$ …)
	$\bar{x} = 13{\cdot}4/8$ *or* $1{\cdot}67[5]$, $\bar{y} = 2{\cdot}02$ (all to 3 s.f.)	**B1**	Find sample means (values to 3 s.f. throughout)
	$s_X^2 = (24{\cdot}7 - 13{\cdot}4^2/8) / 7$ $= 451/1400$ *or* $0{\cdot}3221$ *or* $0{\cdot}5678^2$ *and* $s_Y^2 = (44{\cdot}6 - 20{\cdot}2^2/10) / 9$ $= 949/2250$ *or* $0{\cdot}4218$ *or* $0{\cdot}6494^2$	**M1**	Estimate or imply popln. variances (allow biased here: $0{\cdot}2819$ *or* $0{\cdot}5309^2$) (allow biased here: $0{\cdot}3796$ *or* $0{\cdot}6161^2$)
	$s^2 = (7\,s_X^2 + 9\,s_Y^2) / 16$ (AEF) *or* $(24{\cdot}7 - 13{\cdot}4^2/8 + 44{\cdot}6 - 20{\cdot}2^2/10) / 16$	**M1 A1**	Estimate (pooled) common variance (note s_X^2 and s_Y^2 not needed explicitly)
	$= 6051/16\,000$ *or* $0{\cdot}3782$ *or* $0{\cdot}6150^2$	**A1**	
	$t_{16,\,0.95} = 1{\cdot}746$	***B1**	State or use correct tabular t value
	[–] $t = (\bar{y} - \bar{x}) / s\,\sqrt{(1/8 + 1/10)} = 1{\cdot}18$	**M1 A1**	Find value of t (or can compare $\bar{y} - \bar{x} = 0{\cdot}345$ with $0{\cdot}509$)
	$t < 1{\cdot}75$ so mean masses are the same (AEF)	**DB1 FT**	Correct conclusion (FT on t, dep ***B1**)
	SR: $Z = (\bar{y} - \bar{x}) / \sqrt{(s_X^2/8 + s_Y^2/10)}$ $= 0{\cdot}345 / \sqrt{(0{\cdot}078)} = 1{\cdot}20$	**(B1)**	SR: Implicitly taking s_X^2, s_Y^2 as unequal popln. variances (may also earn first **B1 B1 M1**)
	$Z < 1{\cdot}645$ so mean masses are the same (AEF)	**(B1FT)**	Comparison with $Z_{0.95}$ and conclusion (FT on Z) (can earn at most 5/10)
	Total:	**10**	

Question	Answer	Marks	Guidance
10(i)	$\Sigma x = 20$, $\Sigma y = 30$, $\Sigma xy = 111$, $\Sigma x^2 = 110$, $\Sigma y^2 = 190$ $S_{xy} = 111 - 20 \times 30/5 = -9$ *or* $-1{\cdot}8$ $S_{xx} = 110 - 20^2/5 = 30$ *or* 6 [$S_{yy} = 190 - 30^2/5 = 10$ *or* 2] $b = S_{xy} / S_{xx} = -9/30 = -3/10$ *or* $-0{\cdot}3$	**M1 A1**	Find reqd. values
	$(y - 6) = b\,(x - 4)$, $y = -0{\cdot}3x + 7{\cdot}2$	**M1 A1**	Find gradient b in $y - \bar{y} = b\,(x - \bar{x})$ and hence eqn. of regression line (may be implied by writing $y = a + bx$ and finding a, b)
	Total:	**4**	
10(ii)	$r = S_{xy} / \sqrt{(S_{xx} S_{yy})} = -9 / \sqrt{(30 \times 10)}$	**M1 A1**	Find correlation coefficient r
	$= -0{\cdot}520$	***A1**	
	Total:	**3**	
10(iii)	H_0: $\rho = 0$, H_1: $\rho \neq 0$	**B1**	State both hypotheses (**B0** for r …)
	$r_{5,\,10\%} = 0{\cdot}805$	***B1**	State or use correct tabular two-tail r-value
	Accept H_0 if $\lvert r \rvert <$ tab. value (AEF)	**M1**	State or imply valid method for conclusion
	No [non-zero] correlation (AEF)	**DA1**	Correct conclusion (dep ***A1**, ***B1**)
	Total:	**4**	

Question	Answer	Marks	Guidance
11(a)(i)	$T = 3mg \sin \alpha$ [$= 2mg$]	B1	Find T by resolving forces along plane on P
	$T = kmg\,(5a/4 - a) / a$ [$= ¼\, kmg$]	B1	Find T using Hooke's Law
	$k = 8$	B1	Combine using $\sin \alpha = ⅔$ to find k
	Total:	**3**	
11(a)(ii)	*EITHER:* $\pm\, 2m\, \mathrm{d}^2OQ/\mathrm{d}t^2 = 2mg \sin \alpha - kmg\,(OQ - a) / a$	(M1 A1	Apply Newton's law at general point (e.g. below E)
	$\mathrm{d}^2OQ/\mathrm{d}t^2 = (4g/a)\,(7a/6 - OQ)$	A1	Substitute values of k and $\sin \alpha$
	$\mathrm{d}^2x/\mathrm{d}t^2 = -\,(4g/a)\, x$ where $x = OQ - 7a/6$	A1)	Derive standard SHM form (requires minus sign)
	OR: $2mg \sin \alpha = kmg\,(e - a) / a,\ e = 7a/6$	(M1	Find new equilibrium distance e from O
	$\pm\, 2m\, \mathrm{d}^2x/\mathrm{d}t^2 = 2mg \sin \alpha - kmg\,(e + x - a) / a$	M1 A1	Apply Newton's law at general point (e.g. below E)
	$\mathrm{d}^2x/\mathrm{d}t^2 = -\,(4g/a)\, x$	A1)	Derive standard SHM form (requires minus sign)
	Centre is $7a/6$ (*or* $1{\cdot}17\,a$) from O	B1	State centre of motion
	Period is $\pi\sqrt{(a/g)}$ *or* $0{\cdot}993\sqrt{a}$	B1	State period in simplified form, allowing $g = 10$
	Total:	**6**	
11(a)(iii)	$x_0 = 5a/4 - e = a/12$	B1	Find amplitude x_0 of motion
	$T_{min} = kmg\,(5a/4 - 2x_0 - a) / a = 2\,mg/3$	M1 A1	Find least tension
	$(\mathrm{d}^2x/\mathrm{d}t^2)_{max} = [\pm]\,(4g/a)\, x_0 = [\pm]\, ⅓\, g$	M1 A1	Find maximum acceleration (accepting either sign)
	Total:	**5**	

Question	Answer	Marks	Guidance
11(b)(i)	$x = (1/250)\ \Sigma\ x\ f(x) = 414/250 = 1{\cdot}656$ **AG**	**B1**	Verify given mean
	Total:	**1**	
11(b)(ii)	$p = x/6 = 0{\cdot}276,\ q = 0{\cdot}724$	**M1 A1**	Use $250\ {}^6C_i\ q^{6-i}\ p^i$ and find p <u>and</u> q
	$a = 250\ {}^6C_2\ q^4\ p^2 = 78{\cdot}49 \pm 0{\cdot}01$ (to 2 d.p.)	**A2**	Find either exp. value
	$b = 250\ {}^6C_4\ q^2\ p^4 = 11{\cdot}41 \pm 0{\cdot}01$ (to 2 d.p.)	**A1**	Find other exp. value (deduct single **A1** if either value given to only 1 d.p.)
	Total:	**5**	
11(b)(iii)	H_0: Distribution fits data *or* distribution is binomial (AEF)	**B1**	State (at least) null hypothesis in full Combine values consistent with all exp. values ⩾ 5
	O_i: 48 69 78 32 <u>23</u> E_i: 36·01 82·36 78·49 39·89 <u>13·26</u> (± 0·01)	**M1FT A1**	(FT for **M1** but not **A1** on values of a, b)
	$\chi^2 = 3{\cdot}992 + 2{\cdot}167 + 0{\cdot}003 + 1{\cdot}561 + 7{\cdot}154$	**M1**	Find χ^2
	= 14·9	**A1**	
	No. n of cells: 7 6 <u>5</u> 4 3 $\chi_{n-2,\,0.99}{}^2$: 15·09 13·28 <u>11·34</u> 9·210 6·635	**B1FT**	State or use consistent tabular value $\chi_{n-2,\,0.99}{}^2$ (to 3 s.f.) [FT on number, n, of cells used to find χ^2]
	Accept H_1 if χ^2 > tabular value (AEF) 14·9 [± 0·1] > 11·34 so distn. doesn't fit [data]	**M1**	State or imply valid method for conclusion Conclusion (requires both values correct)
	or manager's belief not justified (AEF)	**A1**	
	Total:	**8**	